Ciencia e Ingeniería
Revista Interdisciplinar de Estudios en Ciencias Básicas e Ingenierías.
Año 2017, Julio- Diciembre, Vol. (4) N° (2) ISSN 2389-9484.
Universidad de La Guajira, Facultades de Ciencias Básicas y Aplicadas e Ingeniería.
La Guajira-Colombia.
Revista en Línea http://revistas.uniguajira.edu.co/index.php/cei

HACIA UNA SOLUCION DEL PROBLEMA DE TIEMPO DE TRANSICION PARA UN SISTEMA DINAMICO MONOMIAL BOOLEANO

TOWARDS A SOLUTION OF THE TRANSIENT PROBLEM FOR BOOLEAN MONOMIAL DYNAMICAL SYSTEMS

Xavier Terán Batista

Candidato a Maestría en Ciencias en Matemáticas de la Universidad de Puerto Rico
Mayagüez, xavier.teran@upr.edu .

Recibido: Enero 20 de 2017 Aceptado: Mayo 15 de 2017

RESUMEN

Los Sistemas Dinámicos Finitos tienen muchas aplicaciones en las Ingenierías y en Ciencias, incluyendo Biología, Ciencias de Cómputos y Ciencias Sociales. En todas esas aplicaciones un problema de interés es determinar cuándo tales sistemas alcanzan el equilibrio; es decir, bajo cuales condiciones es un sistema de punto fijo. Por otra parte, dado un sistema de punto fijo, cuanta cantidad de pasos son requeridos para alcanzar el punto fijo; es decir, ¿Cuál es su tiempo de transición? Bollman y Colon han mostrado que un Sistema Dinámico Monomial Booleano (SDMB) f es un sistema de punto fijo si y solo si cada componente fuertemente conecto del grafo de dependencia Gfde f es primitivo y en efecto, cuando Gf es fuertemente conecto, el tiempo de transición de f es igual a el exponente de Gf. Además, cada grafo dirigido da lugar a un único SDMB y por tanto todo ejemplo de un grafo primitivo con exponente conocido provee un ejemplo de un SDMB de punto fijo con tiempo de transición conocido. Desafortunadamente, el problema general de determinar el exponente de un grafo primitivo es abierto. En este trabajo se muestran varias familias de grafos primitivos para las cuales se puede determinar el exponente y por tanto el tiempo de transición de los correspondientes SDMB.

Palabras clave: SDMB, Tiempo de Transición y Dinámico

ABSTRACT

Finite Dynamical systems have many applications at the engineer and the sciences, including Biology, Computer sciences and Social sciences. In all of this applications a problem of interest in finite dynamical systems is to determine when such a system reaches equilibrium, i.e., under what conditions is it a fixed point system. Moreover, given a fixed point system, how many time steps are required to reach a fixed point, i.e., what is its transient? Bollman and Colón have shown that a Boolean Monomial Dynamical System (BMDS) f is a fixed point system if and only if every strongly connected component of the dependency graph $Gf$of f is primitive and in fact, when Gf is strongly connected, the transient of f is equal to the exponent of Gf.
Furthermore, every directed graph gives rise to a unique BMDS and hence every example of a primitive graph with known exponent gives us an example of a fixed point BMDS with known transient. Unfortunately, the general problem of determining the exponent of a primitive graph is unsolved. In this work we give several families of primitive graphs for which we can determine the exponent and hence the transient of the corresponding BMDS.

Keywords: BMDS, Transition Time and Dynamic

A finite dynamical system (FDS) is a function f: $X \to X$, where X is a finite set and these systems are time discrete. Known examples include cellular automata and Boolean networks, which have found broad applications in engineering, computer science, and, more recently, computational biology.

More general multi-state systems have been used in control theory, the design and analysis of computer simulations, and in computational biology.

In Bollman-Colón (2012), talk about the importance of these systems in genetic modeling and their ability to model the dynamics of gene expressions and relations among genes. This approach enables geneticists to determine the long term impact of a gene on the other genes, see (Colon et al., 2006). Dynamical systems over the field with two elements can be used to model Boolean networks which have applications in both cellular automata and computational biology, see.

B. Elspas (1959), also mentions in applications of linear dynamical systems (LDS) in computer control circuits and communications systems. Some of these applications reach a point in time where they do not experience a change in the state they are in.

Dynamical systems that model such phenomena are said to reach a steady state or fixed point (FP); in all of these applications, an important problem is to give sufficient conditions for a system to be a fixed point system (FPS) and in such a case, to determine the maximum number of time steps necessary to reach a FP, i.e. the transient. Although various authors have given conditions for the existence of or for the number of fixed points systems, little is known about transients for such systems.

Every linear system over a finite field can be represented as a matrix A and the state space structure of f can then be determined by finding the factorization of the characteristic polynomial of A. Bollman-Colón in (2012), establish that if f is a LDS, f: $Zqn \to Zqn$ then f is a FPS if and only if the characteristic polynomial of f is of the form $xi\ (x-1)$ or simply xi.

In Colon et al., (2004), describes a nonlinear system called a boolean monomial dynamical system (BMDS). He defines a discrete dynamical system f: $F2n \to F2n$ where $F2n$ is the $n-fold$ cartesian product of a finite field with two elements. It is well known that f can be written as f = ($f1,2,...,fn$) where each fi is a polynomial. The dynamics of a BMDS f is encoded in its state space (f), which is a directed graph defined as follows. The vertices of (f) are the $2n$ elements of $F2n$. There is a directed edge (a,) in $S(f)$ if $f(a)= b$. In particular, a directed edge from a vertex to itself is admissible. That is, (f)encodes all state transitions of f, and has the property that every vertex has outdegree exactly equal to 1.

Every BMDS has an associated dependency graph G, whose vertices 1,2,...,n correspond to $f1$,$f2$,..., fn. There is a directed edge from i to j if xj divides fi.

The main result in Colon et al., (2004), shows that the structure of the cycles of (f) can be determined from the dependency graph. A principal role is played by strongly connected graphs, that is, directed graphs in which there is a walk between any two vertices. For such a case, Colón defines the loop number to be the minimum positive difference of lengths of circuits through the same vertex. The dependency graph can be decomposed into strongly connected components.

Colon et al., (2004), proves that if the loop number of each strongly connected component of the dependency graph of a BMDS is 1, then f is a FPS.

Furthermore, the loop number of each strongly connected component is equal to the greatest common divisor of the cycle lengths.

Another very important result was proved by Bollman and Colón that says that a BMDS f with a strongly connected dependency graph Gf is a FPS and its only fixed points are (0,0,...,0) and (1,1,...,1) if and only if Gf is primitive; that is, if there exist a positive integer k such that for any pair of vertices (i,j) of G there is a walk of length k from i to j. The smallest positive integer k is called the exponent of G (Bollman etr al., 2007).

It turns out that the exponent of a primitive dependency graph is precisely the transient of the corresponding BMDS. This implies that the problem of determining the transient of a fixed point BMDS reduces to the problem of determining the exponent of a primitive graph. However, methods for finding the exponent of a primitive graph are known only in special cases.

In 1964, Dulmage and Mendelsohn, introduce a family of graphs with known exponent (Dulmage and Mendelsohn , 1964),. They proved that given positive integers $m1 < m2 <\cdots < mk$ such that $gc(m1,m2,...,mk)$= 1, then

$$(m1,m2,...,mk) + r + 1,$$

is an upper bound for the exponent of a primitive graph G, where the mi are the lengths of the cycles of G, r is the length of the longest shortest walk between two vertices that touch at least one vertex of each cycle and $(m1,m2,...,mk)$ is the Frobenius number, i.e., the largest positive integer that is not a non-negative integer linear combination of the mi. They also proved that for a particular family of graphs the above upper bound is also a lower bound.

The same year, Heap and Lynn (1964), defined a family of graphs called Frobenius graphs , and they proved that the exponent of such a graph is given by

$$(m1,m2,...,mk)+ 2mk - 1.$$

In Bollman and Colón (2012), determined that an upper bound for the exponent of a family of graphs consisting of an increasing chain of cycles of coprime lengths is given by the same formula of Dulmage and Mendelsohn, but although it is suspected that this formula is a lower bound, there isn't an established proof.

BIBLIOGRAPHY

Bollman, O. Colón-Reyes, 2012. A New Characterization of the Frobenius Problem, unpublished manuscript, presented at the ``World Congress of Engineering and Technology" (CET), Beijing, China October,

Bollman, O. Colón-Reyes, and E. Orozco, 2007. Fixed Point in Discrete Models for Regulatory Genetics Networks, EURASIP Journal on Bioinformatics and Systems Biology, vol. 2007, Article ID 97356, 8 pages,

Colón-Reyes, R. Laubenbacher, A. Jarrah, B. Sturmfels. 2006, Monomial Dynamical Systems over Finite Fields. Complex Systems, 16pp. 333-342.

Colón-Reyes, R. Laubenbacher, and B. Pareigis, 2004, Boolean Monomial Dynamical Systems, ”Annals of Combinatorics", 8, pp. 425-439.

Dulmage, N. and Mendelsohn S., 1964, Gaps in the exponent set of primitive matrices, Illinois J. Math., 8, pp. 642-656.

Dulmage A.L, Mendelsohn N.S., 1962, The exponent of a primitive matrices, Canad. Math. Bull., vol. 5 pp. 241-244.

Elspas B.; 1959, The Theory of Autonomous Linear Sequential Networks, IRE Transactions on the Circuit Theory, CT-6, 5 pp. 45-60.

Heap B.R., M.S. Lynn, 1964. A Graph-theoretic Algorithm for the Solution of a Linear Diophantine Problem of Frobenius. Numer. Math. 6, 346-354

R. Hernández Toledo, 2005.Linear Finite Dynamical Systems. Comm. Algebra 33, no. 9, pp. 2977-2989,

Mullen G.L., D. Panario, 2013.Handbook of Finite Fields, Discrete Mathematics and its Applications, Series editor K.H. Rosen, 17.1.2, pp.826-829,

Owens R., 2003.An Algorithm to Solve the Frobenius Problem, Mathematics Magazine, Vol. 76, 4, pp.264-275, Oct.,

Ramírez Alfonsin J.L., 2005.The Diophantine Frobenius Problem, Oxford Lecture Series in Mathematics and its Applications 30, Oxford University Press Inc., New York

Rosenblatt, D., 1957, On the Graph and Asymptotic Forms of Finite Boolean Relation Matrices. Naval. Res. Quart., 4151-167.

Tomaiconza O., 2010.Cu\~{n}as en Sistemas Dinámicos, Tesis de Maestría en Matemáticas Puras, Universidad de Puerto Rico en Mayagüez, Mayagüez Puerto Rico,

J. Xu, 2003.Theory and Application of Graphs, Department of Mathematics in University of Science and Technology of China Hefei, Anhui. Network Theory and Applications Vol.10, pp. 70-78. Discrete Visualizer of Dynamics Web Interface, http://dvd.vbi.vt.edu/cgi-bin/git/dvd.pl